DEALING WITH

Math Club 12/19/2011

WHY?

- What is $\infty + \infty$? Is it ∞?

- Then $2\infty = \infty$ and $2 = 1$.

- What is $\infty - \infty$? Is it ∞? Is it $-\infty$? Is it zero?

- What is $\infty \cdot \infty$? Then what's ∞^{∞}? How about $\det \begin{bmatrix} 0 & \infty \\ -\infty & \frac{1}{\infty} \end{bmatrix}$?

- Aaaaaaaaaaaaaaaarrrrrrrrrrrrggggggggggghhhhhhhhh

- There's a reason why all of these are undefined!

BLAKE'S HOTEL

- Blake is the owner and manager of the infinitely large Convenient Plot Hotel.

- The hotel is all filled up with infinitely many guests, making Blake infinitely rich.

- The hotel has an infinite number of rooms, numbered 1, 2, 3…

BLAKE'S FIRST PROBLEM

- But one day, a new guest arrives at the hotel.

- "Oh no," says Blake. "The hotel is already full!"

- But then, being a brilliant mathematician, Blake realizes that there is a way to arrange a room for the new guest!

- He orders Guest 1 to move across to Room 2, Guest 2 to move to Room 3, and so on.

- Now, Room 1 is empty!

BLAKE'S SECOND PROBLEM

- All goes well: Blake is able to accommodate many, many new guests (although his infinitely many guests are infinitely annoyed at having to move all the time). Until one day…

- An infinitely long tour bus shows up with an infinite number of tourists in it.

- "Uh oh," says Blake. "If I move every guest down one, there are still an infinite number of tourists on the bus!"

BLAKE'S SECOND PROBLEM (CONTINUED)

- But fortunately, Blake comes up with another brilliant solution.

- He moves Guest 1 into Room 2, Guest 2 into Room 4, Guest 3 into Room 6, …

- The odd rooms are now empty!

- There are infinitely many odd numbers, hence infinitely many rooms for all the guests on the tour bus.

- Everyone is happy.

BLAKE'S THIRD PROBLEM

- One unfortunate day, an infinite number of infinitely long tour buses arrive at Blake's hotel.

- "Uh oh," says Blake. "I can't do what I just did: after accommodating one infinite bus of tourists, there will still be an infinite number of infinite tour buses!"

- But yet again, Blake comes up with another brilliant solution.

BLAKE'S THIRD PROBLEM (CONTINUED)

- He gets all the buses to line up in his infinite parking lot.

- Then, going down the diagonal, he numbers the guests 1, 3, 5, 7, 9 … until each guest has a number.

- Then he moves every existing guest in the same way as in the second problem.

- Each new guest then settles into the room with his assigned number. Yay!

WHAT HAVE WE LEARNED?

- Infinity does really weird stuff.

- So don't add infinity to infinity. Just call it undefined.

- Or if you must, at least use a limit!